

Name: _____

Date: _____

A2CC: Systems of Linear Equations

Systems of equations, or more than one equation, arise frequently in mathematics. To **solve a system** means to find all sets of values that simultaneously make all equations true. Of special importance are systems of linear equations. You have solved them in your last two Common Core math courses, but we will add to their complexity in this lesson.

Solve the following systems of equations algebraically by either substitution or elimination:

1.
$$\begin{aligned} 2x - y &= -1 \\ 2x + y &= -7 \end{aligned}$$

2.
$$\begin{aligned} 2x + 2y &= 3 \\ x &= 4y - 1 \end{aligned}$$

3.
$$\begin{aligned} x - 2y &= 3 \\ -2x + 4y &= 1 \end{aligned}$$

4.
$$\begin{aligned} 2x - y &= 1 \\ 4x - 2y &= 2 \end{aligned}$$

5.
$$\begin{aligned} 3x + 2y &= 2 \\ 5x + 7y &= -4 \end{aligned}$$

6.
$$\begin{aligned} 3x + 2y &= -9 \\ 2x + y &= -7 \end{aligned}$$

Summary

For a system of linear equations, there can be:

1.

2.

3.

You should be very familiar with solving two-by-two systems of linear equations (two equations and two unknowns). In this lesson, we will extend the method of **elimination** to linear systems of three equations and three unknowns. These linear systems serve as the basis for a field of math known as **Linear Algebra**.

7. Consider the three-by-three system of linear equations shown below. Each equation is numbered in this first exercise to help keep track of our manipulations.

$$(1) \quad 2x + y + z = 15$$

$$(2) \quad 6x - 3y - z = 35$$

$$(3) \quad -4x + 4y - z = -14$$

- (a) The **addition property of equality** allows us to add two equations together to produce a third valid equation. Create a system by adding equations (1) and (2) and (1) and (3). Why is this an effective strategy in this case?
- (b) Use this new two-by-two system to solve the three-by-three.

Just as with two by two systems, sometimes three-by-three systems need to be manipulated by the **multiplication property of equality** before we can eliminate any variables.

8. Consider the system of equations shown below. Answer the following questions based on the system.

$$4x + y - 3z = -6$$

$$-2x + 4y + 2z = 38$$

$$5x - y - 7z = -19$$

(a) Which variable will be easiest to eliminate? Why? Use the multiplicative property of equality and elimination to reduce this system to a two-by-two system.

(b) Solve the two-by-two system from (a) and find the final solution to the three-by-three system.

9. Solve the system of equations shown below. Show each step in your solution process.

$$4x - 2y + 3z = 23$$

$$x + 5y - 3z = -37$$

$$-2x + y + 4z = 27$$

These are challenging problems only because they are long. Be careful and you will be able to solve each one of these more complex systems.

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HOMEWORK

FLUENCY

1. The sum of two numbers is 5 and the larger difference of the two numbers is 39. Find the two numbers by setting up a system of two equations with two unknowns and solving algebraically.

2. Algebraically, find the intersection points of the two lines whose equations are shown below.

$$4x + 3y = -13$$

$$y = 6x - 8$$

3. Show that $x = 10$, $y = 4$, and $z = 7$ is a solution to the system below *without* solving the system formally.

$$x + 2y + z = 25$$

$$4x - y - 5z = 1$$

$$-2x - y + 8z = 32$$

4. In the following system, the value of the constant c is unknown, but it is known that $x = -8$ and $y = 4$ are the x and y values that solve this system. Determine the value of c . Show how you arrived at your answer.

$$-5x + 2y + 3z = 81$$

$$x - y + z = -1$$

$$2x - y + cz = 35$$

5. Solve the following system of equations. Carefully show how you arrived at your answers.

$$\begin{aligned}4x + 2y - z &= 21 \\ -x - 2y + 2z &= 13 \\ 3x - 2y + 5z &= 70\end{aligned}$$

6. Algebraically solve the following system of equations. There are two variables that can be readily eliminated, but your answers will be the same no matter which you eliminate first.

$$\begin{aligned}2x + 5y - z &= -35 \\ x - 3y + 4z &= 31 \\ -3x + 2y + 2z &= -23\end{aligned}$$

7. Algebraically solve the following system of equations. This system will take more manipulation because there are no variables with coefficients equal to 1.

$$\begin{aligned}2x + 3y - 2z &= 33 \\ 4x + 5y + 3z &= 54 \\ -6x - 2y - 8z &= -50\end{aligned}$$